The exterior Steklov problem: formulations, asymptotic behavior, and applications

by Denis S. Grebenkov
LPMC, CNRS – Ecole Polytechnique, IPP, France
Email: denis.grebenkov@polytechnique.edu

In this talk, we present recent advances on the Steklov spectral problem in the complement $\Omega = R^d \backslash \Omega_0$ of a compact set Ω_0 that consists in finding eigenpairs $\{\mu, V\}$ satisfying $\Delta V = 0$ in Ω, with the Robin-type boundary condition $\partial_n V = \mu V$ on a Lipschitz boundary $\partial\Omega$. We first overview several distinct formulations of this exterior problem and discuss their equivalence. We then switch to the generalized Steklov problem, in which $\Delta V = 0$ is replaced by $(\Delta - p)V = 0$, with a parameter $p > 0$. We obtain the asymptotic behavior of the Steklov eigenvalues as $p \to 0$ and reveal their non-analytical dependence on p that depends on the space dimension. Several applications of these asymptotic results to reflected diffusion in chemical physics and statistics physics are presented. Some open problems and further perspectives are discussed.

References:

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