A class of parabolic fractional reaction-diffusion systems with control of total mass: Theory and numerics

Maha DAOUD

Abstract

In this talk based on [1, 2], we present some new results about global-in-time existence of strong solutions to a class of parabolic fractional reaction–diffusion systems posed in a bounded open subset of $\mathbb{R}^d$. The nonlinear reactive terms are assumed to satisfy natural structure conditions which provide nonnegativity of the solutions and uniform control of the total mass. The diffusion operators are of type $u_i \mapsto d_i(-\Delta)^{s_i}u_i$ where $0 < s_i < 1$. For more details about this kind of operators, we refer the interested reader to [3] and references therein. Global existence of strong solutions is proved under the assumption that the reactive terms are at most of polynomial growth. Our results extend previous results obtained in [4, 5] where the diffusion operators are of type $u_i \mapsto -d_i\Delta u_i$.

Also, we present some numerical simulations in order to examine the global existence of solutions to systems with exponentially growing right-hand sides, which remains so far an open theoretical question even in the case where the diffusion is driven by the classical Laplacian. See [6, 7, 8].

References

[1] M. Daoud, E.-H. Laamri and A. Baalal, *A class of parabolic fractional reaction-diffusion systems with control of total mass: Theory and numerics*, J. Pseudo-Differ. Oper. Appl. 15(18) (2024). DOI : 10.1007/s11868-023-00576-w.

[2] M. Daoud, *A class of parabolic reaction-diffusion systems governed by spectral fractional Laplacians : Analysis and numerical simulations*, Submitted. ArXiv-2502.13771 (2025).

[3] M. Daoud and E.-H. Laamri, *Fractional Laplacians : a short survey*, Discrete Contin. Dyn. Syst. - S, 15(1) (2022), 95–116.

[4] E.-H. Laamri, *Global existence of classical solutions for a class of reaction-diffusion systems*, Acta. Appl. Math. 115 (2011), 153-165.

[5] M. Pierre, *Global Existence in Reaction-Diffusion Systems with Control of Mass : a Survey*, Milan J. Math. 78 (2010), 417–455.

[6] A. Haraux and A. Youkana, *On a result of K. Masuda concerning reaction-diffusion equations*, Tôhoku Math. J. 40 (1988), 159–163.

[7] A. Barabanova, *On the global existence of solutions of a reaction-diffusion equation with exponential nonlinearity*, Proc. Amer. Math. Soc. 122 (1994), 827–831.

[8] M.A. Herrero, A.A. Lacey and J.L. Velàzquez, *Global existence for reaction-diffusion systems modelling ignition*, Arch. Rat. Mech. Anal., 142 (1998), 219–251.